General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MPC4

Unit Pure Core 4

Monday 10 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 (a) (i) Express $\dfrac{5-8x}{(2+x)(1-3x)}$ in the form $\dfrac{A}{2+x}+\dfrac{B}{1-3x}$, where A and B are integers. *(3 marks)*

(ii) Hence show that $\displaystyle\int_{-1}^{0} \frac{5-8x}{(2+x)(1-3x)}\,\mathrm{d}x = p\ln 2$, where p is rational. *(4 marks)*

(b) (i) Given that $\dfrac{9-18x-6x^2}{2-5x-3x^2}$ can be written as $C+\dfrac{5-8x}{2-5x-3x^2}$, find the value of C. *(1 mark)*

(ii) Hence find the exact value of the area of the region bounded by the curve $y=\dfrac{9-18x-6x^2}{2-5x-3x^2}$, the x-axis and the lines $x=-1$ and $x=0$.

You may assume that $y>0$ when $-1 \leqslant x \leqslant 0$. *(2 marks)*

---

2 The acute angles α and β are given by $\tan\alpha = \dfrac{2}{\sqrt{5}}$ and $\tan\beta = \frac{1}{2}$.

(a) (i) Show that $\sin\alpha = \frac{2}{3}$, and find the exact value of $\cos\alpha$. *(2 marks)*

(ii) Hence find the exact value of $\sin 2\alpha$. *(2 marks)*

(b) Show that the exact value of $\cos(\alpha-\beta)$ can be expressed as $\frac{2}{15}(k+\sqrt{5})$, where k is an integer. *(4 marks)*

---

3 (a) Find the binomial expansion of $(1+6x)^{-\frac{1}{3}}$ up to and including the term in x^2. *(2 marks)*

(b) (i) Find the binomial expansion of $(27+6x)^{-\frac{1}{3}}$ up to and including the term in x^2, simplifying the coefficients. *(3 marks)*

(ii) Given that $\sqrt[3]{\dfrac{2}{7}} = \dfrac{2}{\sqrt[3]{28}}$, use your binomial expansion from part **(b)(i)** to obtain an approximation to $\sqrt[3]{\dfrac{2}{7}}$, giving your answer to six decimal places. *(2 marks)*

4 A curve is defined by the parametric equations $x = 8e^{-2t} - 4$, $y = 2e^{2t} + 4$.

(a) Find $\dfrac{dy}{dx}$ in terms of t. *(3 marks)*

(b) The point P, where $t = \ln 2$, lies on the curve.

(i) Find the gradient of the curve at P. *(1 mark)*

(ii) Find the coordinates of P. *(2 marks)*

(iii) The normal at P crosses the x-axis at the point Q. Find the coordinates of Q. *(3 marks)*

(c) Find the Cartesian equation of the curve in the form $xy + 4y - 4x = k$, where k is an integer. *(3 marks)*

---

5 The polynomial $f(x)$ is defined by $f(x) = 4x^3 - 11x - 3$.

(a) Use the Factor Theorem to show that $(2x + 3)$ is a factor of $f(x)$. *(2 marks)*

(b) Write $f(x)$ in the form $(2x + 3)(ax^2 + bx + c)$, where a, b and c are integers. *(2 marks)*

(c) (i) Show that the equation $2\cos 2\theta \sin\theta + 9\sin\theta + 3 = 0$ can be written as $4x^3 - 11x - 3 = 0$, where $x = \sin\theta$. *(3 marks)*

(ii) Hence find all solutions of the equation $2\cos 2\theta \sin\theta + 9\sin\theta + 3 = 0$ in the interval $0° < \theta < 360°$, giving your solutions to the nearest degree. *(4 marks)*

6 The points A, B and C have coordinates $(3, -2, 4)$, $(1, -5, 6)$ and $(-4, 5, -1)$ respectively.

The line l passes through A and has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 7 \\ -7 \\ 5 \end{bmatrix}$.

(a) Show that the point C lies on the line l. *(2 marks)*

(b) Find a vector equation of the line that passes through points A and B. *(3 marks)*

(c) The point D lies on the line through A and B such that the angle CDA is a right angle.

Find the coordinates of D. *(5 marks)*

(d) The point E lies on the line through A and B such that the area of triangle ACE is three times the area of triangle ACD.

Find the coordinates of the two possible positions of E. *(4 marks)*

---

7 The height of the tide in a certain harbour is h metres at time t hours. Successive high tides occur every 12 hours.

The **rate of change** of the height of the tide can be modelled by a function of the form $a\cos(kt)$, where a and k are constants. The largest value of this rate of change is 1.3 metres per hour.

Write down a differential equation in the variables h and t. State the values of the constants a and k. *(3 marks)*

---

8 (a) Find $\int t\cos\left(\frac{\pi}{4}t\right)\,\mathrm{d}t$. *(4 marks)*

(b) The platform of a theme park ride oscillates vertically. For the first 75 seconds of the ride,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{t\cos\left(\frac{\pi}{4}t\right)}{32x}$$

where x metres is the height of the platform above the ground after time t seconds.

At $t = 0$, the height of the platform above the ground is 4 metres.

Find the height of the platform after 45 seconds, giving your answer to the nearest centimetre. *(6 marks)*